ALGEBRA — Answers

1 **a** $= 64$ **b** $= 216$ **c** $= 1$

d $= 625$ **e** $= -243$ **f** $= \frac{1}{16}$

g $= \frac{8}{27}$ **h** $= -\frac{1}{64}$ **i** $= (\frac{4}{3})^2 = \frac{16}{9}$ or $1\frac{7}{9}$

j $= (\frac{3}{2})^4 = \frac{81}{16}$ or $5\frac{1}{16}$ **k** $= 0.000\,01$ **l** $= -0.008$

2 **a** 2^8 **b** 2^7 **c** 2^0 **d** 2^4 **e** 2^9 **f** 2^{14}

3 **a** $= 8p^7$ **b** $= x^{10}$ **c** $= 6n^5$ **d** $= y^{12}$

e $= 2b$ **f** $= p^4q^3$ **g** $= x^3y$ **h** $= 6r^2s^3$

i $= 2x^3y^7$ **j** $= 4a^5b^8$ **k** $= 125r^3s^6 \div 100r^2s^2 = \frac{5}{4}rs^4$ **l** $= 15p^3q$

4 **a** $= \frac{1}{3^2} = \frac{1}{9}$ **b** $= 1$ **c** $= \frac{1}{(-2)^6} = \frac{1}{64}$

d $= 6^2 = 36$ **e** $= (\frac{3}{2})^{-3} = (\frac{2}{3})^3 = \frac{8}{27}$ **f** $= \sqrt{9} = 3$

g $= \sqrt[4]{16} = 2$ **h** $= \sqrt[3]{-27} = -3$ **i** $= \sqrt{\frac{1}{49}} = \frac{1}{7}$

j $= \sqrt[3]{125} = 5$ **k** $= \sqrt{\frac{4}{9}} = \frac{2}{3}$ **l** $= \frac{1}{\sqrt{36}} = \frac{1}{6}$

m $= \frac{1}{\sqrt[4]{81}} = \frac{1}{3}$ **n** $= \frac{1}{\sqrt[3]{-64}} = -\frac{1}{4}$ **o** $= \sqrt[5]{32} = 2$

p $= \sqrt[3]{-\frac{8}{125}} = -\frac{2}{5}$ **q** $= \sqrt{\frac{9}{4}} = \frac{3}{2}$ or $1\frac{1}{2}$ **r** $= (\frac{27}{8})^{-\frac{1}{3}} = \sqrt[3]{\frac{8}{27}} = \frac{2}{3}$

5 **a** $= (\sqrt{4})^3 = 2^3 = 8$ **b** $= (\sqrt[3]{27})^2 = 3^2 = 9$

c $= (\sqrt[4]{16})^3 = 2^3 = 8$ **d** $= (\sqrt[3]{-125})^2 = (-5)^2 = 25$

e $= (\sqrt{9})^5 = 3^5 = 243$ **f** $= \frac{1}{(\sqrt[3]{8})^2} = \frac{1}{2^2} = \frac{1}{4}$

g $= \frac{1}{(\sqrt{36})^3} = \frac{1}{6^3} = \frac{1}{216}$ **h** $= (\sqrt[3]{\frac{1}{8}})^4 = (\frac{1}{2})^4 = \frac{1}{16}$

i $= (\sqrt{\frac{4}{9}})^3 = (\frac{2}{3})^3 = \frac{8}{27}$ **j** $= (\sqrt[3]{216})^2 = 6^2 = 36$

k $= (\sqrt{\frac{16}{9}})^3 = (\frac{4}{3})^3 = \frac{64}{27}$ or $2\frac{10}{27}$ **l** $= (\sqrt[3]{-\frac{27}{64}})^4 = (-\frac{3}{4})^4 = \frac{81}{256}$

m $= \sqrt{\frac{4}{100}} = \frac{2}{10} = \frac{1}{5}$ or 0.2 **n** $= (\frac{9}{4})^{-\frac{3}{2}} = (\sqrt{\frac{4}{9}})^3 = (\frac{2}{3})^3 = \frac{8}{27}$

o $= (\sqrt[3]{\frac{64}{1000}})^2 = (\frac{4}{10})^2 = \frac{4}{25}$ or 0.16 **p** $= (\frac{25}{16})^{-\frac{3}{2}} = (\sqrt{\frac{16}{25}})^3 = (\frac{4}{5})^3 = \frac{64}{125}$

q $= (\frac{81}{16})^{\frac{3}{4}} = (\sqrt[4]{\frac{81}{16}})^3 = (\frac{3}{2})^3 = \frac{27}{8}$ or $3\frac{3}{8}$ **r** $= (\frac{64}{27})^{-\frac{4}{3}} = (\sqrt[3]{\frac{27}{64}})^4 = (\frac{3}{4})^4 = \frac{81}{256}$

6

a $= \sqrt{4} \times \sqrt[3]{27}$ $= 2 \times 3 = 6$

b $= \sqrt[4]{16} + \sqrt{25}$ $= 2 + 5 = 7$

c $= \frac{1}{\sqrt[3]{8}} \div \sqrt{36}$ $= \frac{1}{2} \div 6 = \frac{1}{12}$

d $= \sqrt[3]{-64} \times (\sqrt{9})^3$ $= -4 \times 27 = -108$

e $= 3^2 - \sqrt[3]{-8}$ $= 9 - (-2) = 11$

f $= \sqrt{\frac{1}{25}} \times 4^2$ $= \frac{1}{5} \times 16 = \frac{16}{5}$ or $3\frac{1}{5}$

g $= (\sqrt[4]{81})^3 - \sqrt{49}$ $= 27 - 7 = 20$

h $= \sqrt[3]{27} \times (\sqrt{\frac{9}{4}})^3$ $= 3 \times \frac{27}{8} = \frac{81}{8}$ or $10\frac{1}{8}$

i $= \sqrt{9} \times (\sqrt[5]{-32})^3$ $= 3 \times (-8) = -24$

j $= \sqrt{121} + \sqrt[5]{32}$ $= 11 + 2 = 13$

k $= \sqrt{100} \div (\sqrt{\frac{1}{4}})^3$ $= 10 \div \frac{1}{8} = 80$

l $= \frac{1}{\sqrt[4]{16}} \times (\sqrt[5]{243})^2$ $= \frac{1}{2} \times 9 = \frac{9}{2}$ or $4\frac{1}{2}$

7

a $= x^2$

b $= y^{-6}$

c $= 3p^{-4}$

d $= 8x^{-12}$

e $= y^{\frac{5}{2}}$

f $= 8b^{\frac{2}{3} + \frac{1}{4}} = 8b^{\frac{11}{12}}$

g $= x^{\frac{3}{5} - \frac{1}{3}} = x^{\frac{4}{15}}$

h $= a^{\frac{1}{2} - \frac{4}{3}} = a^{-\frac{5}{6}}$

i $= p^{\frac{1}{4} - (-\frac{1}{5})} = p^{\frac{9}{20}}$

j $= 9x^{\frac{4}{5}}$

k $= y^{1 + \frac{5}{6} - \frac{3}{2}} = y^{\frac{1}{3}}$

l $= \frac{1}{3}t$

m $= b^{2 + \frac{1}{4} - \frac{1}{2}} = b^{\frac{7}{4}}$

n $= y^{\frac{1}{2} + \frac{1}{3} - 1} = y^{-\frac{1}{6}}$

o $= 2x^{\frac{2}{3} + (-\frac{1}{6}) - \frac{3}{4}} = 2x^{-\frac{1}{4}}$

p $= \frac{1}{4}a^{1 + \frac{3}{4} - (-\frac{1}{2})} = \frac{1}{4}a^{\frac{9}{4}}$

8

a $x = 6^2 = 36$

b $x = 5^3 = 125$

c $x^{\frac{1}{2}} = \frac{1}{2}$ $x = (\frac{1}{2})^2 = \frac{1}{4}$

d $x^{\frac{1}{4}} = 3$ $x = 3^4 = 81$

e $x^{\frac{1}{2}} = \sqrt[3]{8} = 2$ $x = 2^2 = 4$

f $x^{\frac{1}{3}} = \pm\sqrt{16} = \pm 4$ $x = (\pm 4)^3 = \pm 64$

g $x^{\frac{1}{3}} = \pm\sqrt[4]{81} = \pm 3$ $x = (\pm 3)^3 = \pm 27$

h $x^{\frac{3}{2}} = \frac{1}{27}$ $x^{\frac{1}{2}} = \sqrt[3]{\frac{1}{27}} = \frac{1}{3}$ $x = (\frac{1}{3})^2 = \frac{1}{9}$

9

a $= x^{\frac{1}{2}}$

b $= x^{-\frac{1}{3}}$

c $= x^2 \times x^{\frac{1}{2}} = x^{\frac{5}{2}}$

d $= \frac{x^{\frac{1}{4}}}{x} = x^{-\frac{3}{4}}$

e $= (x^3)^{\frac{1}{2}} = x^{\frac{3}{2}}$

f $= x^{\frac{1}{2}} \times x^{\frac{1}{3}} = x^{\frac{5}{6}}$

g $= (x^{\frac{1}{2}})^5 = x^{\frac{5}{2}}$

h $= x^{\frac{2}{3}} \times x^{\frac{3}{2}} = x^{\frac{13}{6}}$

10

a $4x^{-\frac{1}{2}}$

b $\frac{1}{2}x^{-1}$

c $\frac{3}{4}x^{-3}$

d $\frac{1}{9}x^{-2}$

e $\frac{2}{5}x^{-\frac{1}{3}}$

f $\frac{1}{3}x^{-\frac{3}{2}}$

11

a $= (2^3)^2 = 2^6$

b $= (2^{-2})^{-2} = 2^4$

c $= (2^{-1})^{\frac{1}{3}} = 2^{-\frac{1}{3}}$

d $= (2^4)^{-\frac{1}{6}} = 2^{-\frac{2}{3}}$

e $= (2^3)^{\frac{2}{5}} = 2^{\frac{6}{5}}$

f $= (2^{-5})^{-3} = 2^{15}$

12

a $= (3^2)^x = 3^{2x}$

b $= (3^4)^{x+1} = 3^{4x+4}$

c $= (3^3)^{\frac{x}{4}} = 3^{\frac{3}{4}x}$

d $= (3^{-1})^x = 3^{-x}$

e $= (3^2)^{2x-1} = 3^{4x-2}$

f $= (3^{-3})^{x+2} = 3^{-3x-6}$

13

a $= 2 \times 2^x = 2y$

b $= 2^{-2} \times 2^x = \frac{1}{4}y$

c $= (2^x)^2 = y^2$

d $= (2^3)^x = 2^{3x} = (2^x)^3 = y^3$

e $= 2^3 \times 2^{4x} = 8y^4$

f $= (2^{-1})^{x-3} = 2^3 \times 2^{-x} = \frac{8}{y}$

14

a $2^x = 2^6$
$x = 6$

b $5^{x-1} = 5^3$
$x - 1 = 3$
$x = 4$

c $3^{x+4} = 27 = 3^3$
$x + 4 = 3$
$x = -1$

d $(2^3)^x = 2^{3x} = 2$
$3x = 1$
$x = \frac{1}{3}$

e $3^{2x-1} = 3^2$
$2x - 1 = 2$
$x = \frac{3}{2}$

f $16 = 4^2 = 4^{3x-2}$
$2 = 3x - 2$
$x = \frac{4}{3}$

g $(3^2)^{x-2} = 3^{2x-4} = 3^3$
$2x - 4 = 3$
$x = \frac{7}{2}$

h $(2^3)^{2x+1} = 2^{6x+3} = 2^4$
$6x + 3 = 4$
$x = \frac{1}{6}$

i $(7^2)^{x+1} = 7^{2x+2} = 7^{\frac{1}{2}}$
$2x + 2 = \frac{1}{2}$
$x = -\frac{3}{4}$

j $3^{3x-2} = (3^2)^{\frac{1}{3}} = 3^{\frac{2}{3}}$
$3x - 2 = \frac{2}{3}$
$x = \frac{8}{9}$

k $(6^{-1})^{x+3} = 6^{-x-3} = 6^2$
$-x - 3 = 2$
$x = -5$

l $(2^{-1})^{3x-1} = 2^{1-3x} = 2^3$
$1 - 3x = 3$
$x = -\frac{2}{3}$

15

a $2^{x+3} = (2^2)^x = 2^{2x}$
$x + 3 = 2x$
$x = 3$

b $5^{3x} = (5^2)^{x+1} = 5^{2x+2}$
$3x = 2x + 2$
$x = 2$

c $(3^2)^{2x} = 3^{4x} = 3^{x-3}$
$4x = x - 3$
$x = -1$

d $(4^2)^x = 4^{2x} = 4^{1-x}$
$2x = 1 - x$
$x = \frac{1}{3}$

e $(2^2)^{x+2} = (2^3)^x$
$2^{2x+4} = 2^{3x}$
$2x + 4 = 3x$
$x = 4$

f $(3^3)^{2x} = (3^2)^{3-x}$
$3^{6x} = 3^{6-2x}$
$6x = 6 - 2x$
$x = \frac{3}{4}$

g $6^{3x-1} = (6^2)^{x+2}$
$6^{3x-1} = 6^{2x+4}$
$3x - 1 = 2x + 4$
$x = 5$

h $(2^3)^x = (2^4)^{2x-1}$
$2^{3x} = 2^{8x-4}$
$3x = 8x - 4$
$x = \frac{4}{5}$

i $(5^3)^x = 5^{x-3}$
$5^{3x} = 5^{x-3}$
$3x = x - 3$
$x = -\frac{3}{2}$

j $(3^{-1})^x = 3^{x-4}$
$3^{-x} = 3^{x-4}$
$-x = x - 4$
$x = 2$

k $(2^{-1})^{1-x} = (2^{-3})^{2x}$
$2^{x-1} = 2^{-6x}$
$x - 1 = -6x$
$x = \frac{1}{7}$

l $(2^{-2})^{x+1} = (2^3)^x$
$2^{-2x-2} = 2^{3x}$
$-2x - 2 = 3x$
$x = -\frac{2}{5}$

16

a $= x^3 - 1$

b $= 2x^2 + 6x^3$

c $= 3 - x^2$

d $= 12x^3 + 8x$

e $= 3x^3 + 2x$

f $= 3 - 3x^2$

g $= 5x^{\frac{1}{2}} + x^2$

h $= 3x^2 - x^{-1}$

i $= x^6 + x^4 - 3x^2 - 3$

j $= 2x^9 + 6x^5 + x^5 + 3x$
$= 2x^9 + 7x^5 + 3x$

k $= x^3 - 1 - 2 + 2x^{-3}$
$= x^3 - 3 + 2x^{-3}$

l $= x^3 - x^{\frac{5}{2}} - x^{\frac{5}{2}} + x^2$
$= x^3 - 2x^{\frac{5}{2}} + x^2$

17

a $= x^2 + 2$

b $= 2t^3 - 3t$

c $= x - 3x^{\frac{1}{2}}$

d $= \dfrac{y^5 - 6y^2}{3y}$
$= \frac{1}{3}y^4 - 2y$

e $= p^{\frac{1}{4}} + p^{\frac{3}{4}}$

f $= 2w^{\frac{3}{2}} - \frac{1}{2}w$

g $= \dfrac{x^{\frac{1}{2}}(x+1)}{x+1}$
$= x^{\frac{1}{2}}$

h $= \dfrac{t^{\frac{1}{2}} \times 2t(t^2 - 2)}{t^2 - 2}$
$= 2t^{\frac{3}{2}}$